Chapter 19

FARADAY'S   LAW

A.) Observations and Definitions:

1.) In the Gauss's Law chapter, the idea of flux was introduced as:

$$\Phi_h = \int_S \mathbf{h} \bullet d\mathbf{S},$$

where dS is a differential surface area vector defined on a surface through which the vector-field h passes, and the integral sums all such dot products over the entire surface.

2.) As such, a magnetic field B passing through the face of a coil of area A (see Figure 19.1a) produces a magnetic flux Φ_m through the coil. The amount of flux is dependent upon several parameters:

a.) The area A of the coil's face (as used here, "face" is defined as the area bounded by the coil--the larger the face, the more field lines can pass through);

b.) The magnitude of the magnetic field (it should be obvious that the larger the field, the more field lines will pass through a given face area); and

c.) As Figures 19.1a, b, and c show, the angular relationship between the coil's orientation and the direction of the magnetic field (notice that when the coil is positioned as in Figure 19.1a, there is more magnetic
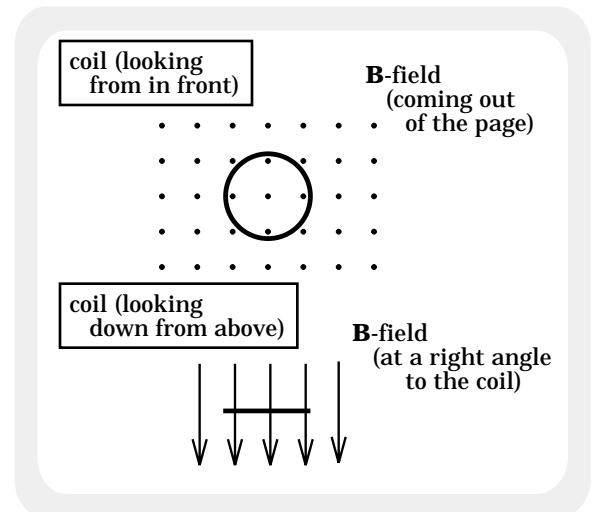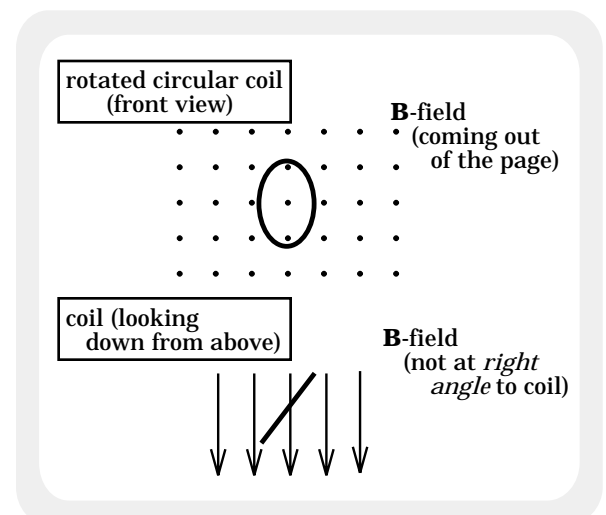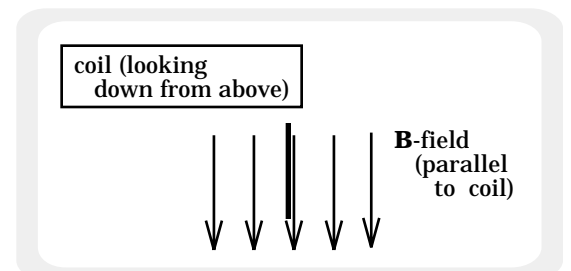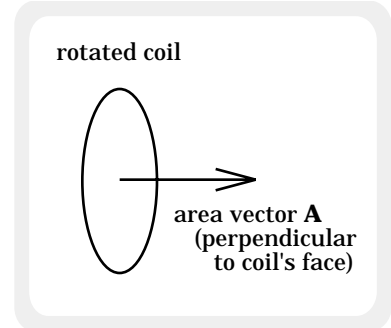

FIGURE 19.1a

FIGURE 19.1b

FIGURE 19.1c

flux passing through its face than is the case in Figure 19.1b; notice also that there is no magnetic flux through the coil in Figure 19.1c).

   d.) This angular variation is taken into account in the following way:

rotated coil

area vector **A**
(perpendicular
to coil's face)

    i.) Define a vector A whose magnitude is the area of the coil's face and whose direction is <u>perpendicularly out</u> from the face (see Figure 19.2). With A so defined (note: this variable will sometimes be denoted as S, standing for surface area--we will use both notations), the magnetic flux through the coil's face is:

$$\Phi_m = B \cdot A.$$

    e.) Example of a flux calculation: Determine the magnetic flux for the situation depicted in Figure 19.3a (the sketch shows a circular coil FROM ABOVE). Assume that the magnetic field intensity is .02 teslas, the coil's radius is .3 meters, and the angle between the hoop itself and the magnetic field is 30^0.
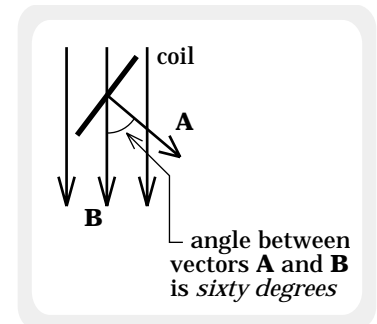

30^0

coil
(from
above)

B-field
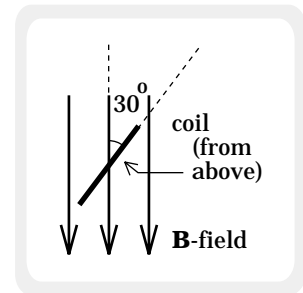
FIGURE 19.3a

    i.) To begin with, notice that the magnitude of the area of the coil's face is:

$$A = \pi r^2,$$

while the angle θ between A and B is 60^0. . . that's right, 60^0! The angle in this dot product is between the <u>line of B</u> and the <u>line of A</u>, where A is a vector PERPENDICULAR to the face of the coil (see Figure 19.3b).

coil

A

B

angle between
vectors **A** and **B**
is *sixty degrees*

FIGURE 19.3b

    ii.) Using our definition for flux:

$$\Phi_m = B \cdot A.$$
$$= \ |B| \qquad |A| \qquad \cos \theta$$
$$= (.02 \text{ T})\ [(\pi)\ (.3 \text{ m})^2]\ \cos (60^0)$$
$$= 2.83 \times 10^{-3} \text{ tesla·meters}.$$

Note: The unit tesla·meters is given a special name--the weber. That means the units for a magnetic field could be written as webers per meter.

3.) Getting a bit more exotic, consider a magnetic field that varies as $B_0\sin(k_1xt)k$, where k_1 is a constant provided to keep the units straight, k is a unit vector in the +z-direction, x is the x-coordinate of a point in question, and t is a time variable for the field (that is, the field not only changes as x gets larger, it also changes with time). Given all this, what is the general expression AS A FUNCTION OF TIME for the magnetic flux through the rectangular coil shown in Figure 19.4a? That is, derive an expression that will allow us to determine the magnetic flux at any instant.
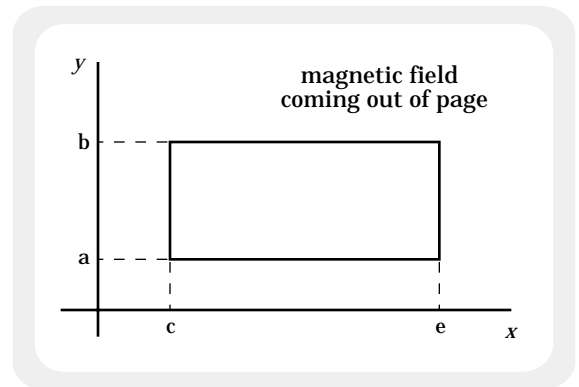

FIGURE 19.4a

a.) At a given instant t, B is not the same from place to place across the coil's face (it varies with x). To deal with this, we must begin with a differentially thin strip of width dx. Because this strip is differential, B will be the same throughout the area defined by the strip. If we can determine the flux through that strip, we can determine the total flux through the coil by summing all such differential flux quantities using integration.
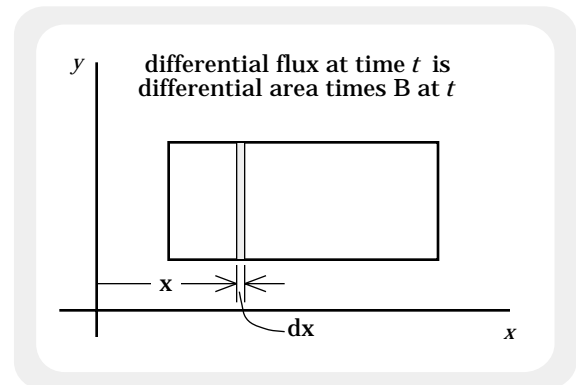
FIGURE 19.4b

b.) The setup is shown in Figure 19.4b. The direction of the differential area is perpendicular to the face (i.e., either into or out of the page). To make things easy, we will define it to be out of the page in the direction of the magnetic field vector.

The magnitude of the differential surface area vector dS is:

$$dS = (height)(width)$$
$$= (b - a)dx.$$

c.) The differential flux is:

$$d\Phi_m = B \cdot dS$$

$$= B(dS)\cos 0^0$$
$$= B[(b - a)dx](1)$$
$$= [B_0\sin(k_1xt)][(b - a)dx].$$

d.) The total flux is:

$$\Phi_m = \int d\Phi_m$$

$$= \int B \bullet dS$$

$$= \int_{x=c}^{e} \left[B_0 \sin(k_1 xt) \right] \left[(b-a)dx \right]$$

$$= B_0 (b-a) \int_{x=c}^{e} \sin(k_1 xt)dx$$

$$= B_0 (b-a) \left[-\frac{1}{k_1 t} \cos(k_1 xt) \right]_{x=c}^{e}$$

$$= B_0 (b-a) \left[-\left[\frac{1}{k_1 t} \cos(k_1 et) - \frac{1}{k_1 t} \cos(k_1 ct) \right] \right].$$

e.) IMPORTANT: This approach is useful only when you are determining the magnetic flux due to a magnetic field that <u>varies from point to point</u> over the face of the coil. The steps involved are:

i.) Find an area in which the magnetic field is the same at every point. This will, in all probability, be a differential area.

ii.) Determine an expression for the magnitude of that differential area dS. This should be in terms of physical parameters along with some differential length (dx, dy, dr, whatever).

iii.) Determine the differential flux, then integrate that expression to determine the total flux through the surface.

f.) IMPORTANT: If B is a constant over the coil's face, always use Φ_m = B·A to determine the magnetic flux through the coil's face.

B.) Faraday's Law:

1.) In 1831, Michael Faraday noticed a peculiar phenomenon. The following scenario and commentary highlight his observations:

a.) Attach a coil to a galvanometer. There will be no current flowing in the coil because there is no power being provided to the coil.

b.) Put the coil in a constant magnetic field (Figure 19.5). There will be a magnetic flux through the coil, but there will still be no current in the coil-- there will still be no power being supplied to the wire loop.

FIGURE 19.5

c.) <u>Change</u> the magnetic flux by either:

i.) Changing the magnetic field strength;

ii.) Changing the area of the coil's face;

iii.) Changing the orientation of the coil relative to the magnetic field (i.e., change the angle θ in the dot product B·A); or

iv.) Some combination thereof.

d.) <u>Observation</u>: Even though there will still be no standard power supply in the circuit, as the magnetic flux changes--and ONLY while the flux changes--a current in the coil will be registered by the galvanometer.

2.) There are many ways we can explain this. From the point of view of the free charge in the wire, decreasing the magnetic field (hence, decreasing the magnetic flux through the coil) is like moving from a region of bigger B-field into a region of smaller B-field. Charges moving in a magnetic field feel a magnetic force defined by qvxB, so free charges in the wire will feel a force that makes them circulate through the coil.
What is important to note is that Faraday did not see the situation in these terms. His evaluation was as follows:

a.) <u>Observation 1</u>: As is stated above, while the magnetic flux changes--and only while the flux changes--a current in the coil is registered by the galvanometer. Therefore, a changing magnetic flux must induce an EMF (like a voltage) in the coil which, in turn, generates current that lasts as long as the flux continues to change.

Note: Electromotive force (EMF) is that part of a power source that motivates charge to move--it is that part that actually puts energy into the system. Its units are volts. You might wonder why this has a special
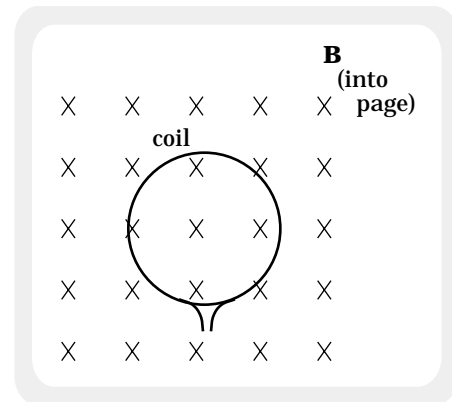
definition. In the past, we oversimplified the workings of power supplies. A battery, for instance, has an internal resistance r. That means the battery heats up taking energy out of the system when current is drawn from it (this internal voltage drop is equal to ir). To technically delineate between the battery's terminal voltage V_o (i.e., the voltage measured across its terminals) and the actual charge-motivating character of the battery, the concept of the electromotive force (EMF) was defined.   In a battery, the EMF = V_o + ir.

The point is that Faraday alluded to a charge-motivating property that seems to exist when a coil of wire is placed in a changing magnetic field. As such, he related charge flow to an induced EMF that, he reasoned, must exist if current was to flow in the circuit.

b.) Observation 2: The induced EMF is related to the rate at which the flux changes and the number of winds N in the coil. Assuming the change is constant, this induced EMF is found to be:

$$\text{induced EMF} = -N \, (\Delta\Phi_m)/(\Delta t)$$

Note 1: The negative sign in front of the expression will be explained later.

Note 2: The symbol most often used for an EMF is ε.

c.) If the flux-change is not constant, the induced EMF becomes:

$$\text{induced EMF} = -N \, (d\Phi_m)/(dt).$$

THIS IS IMPORTANT. The easiest way to do most Faraday's Law problems is to write out Φ_m as it exists at any arbitrary point in time, and then take its time derivative.

d.) Plugging in our defining expression for magnetic flux (i.e., Φ_m = B·A = $|B||A|\cos\theta$), we can rewrite Faraday's Law as:

$$\text{induced EMF} = -N \, (d\Phi_m)/(dt)$$

$$\Rightarrow \quad \varepsilon = -N \, d(BA\cos\theta)/(dt).$$

In this expression, B is the magnetic field intensity, A is the area of the coil, and θ is the angle between the magnetic field vector B and the area vector A (remember, the area vector is directed normal to the face of the coil), all evaluated at an arbirtary point in time.

e.) Even when you are dealing with a simply, constant flux change, you have to think a bit about what the system is doing before you take the derivative. Not clear? Consider:

i.) If B is the only parameter that is varying, the EMF becomes

$\mathcal{E}$ = -N d(BA cos θ)/(dt) = -NAcosθ (dB/dt).

ii.) If A is the only parameter that is varying, the EMF becomes

$\mathcal{E}$ = -N d(BA cos θ)/(dt) = -NBcosθ (dA/dt).

iii.) If both A and B are varying, the EMF becomes

$\mathcal{E}$ = -N d(BA cos θ)/(dt) = -[NAcosθ (dB/dt) + NBcosθ (dA/dt)].

iv.) The form of the derivative depends upon what is changing!

f.) What's even more exciting is the fact that the information required to determine how a parameter changes can come in a number of ways. If the magnetic field is changing, for example:

i.) You could be told that B = 3 tesla at t = 2 seconds and B = 21 teslas at t = 5 seconds. In that case, dB/dt = $(B_2 - B_1)/(t_2 - t_1)$ = (21 teslas - 3 teslas)/(5 seconds - 2 seconds) = 6 teslas/second. OR:

ii.) You could simply be told that B changes at a rate of 6 teslas/second (i.e., you could be given dB/dt straight away).

g.) The moral of the story? Be thoughtful and use your head.

3.) Example: A 20 centimeter radius coil comprised of 25 turns (i.e., 25 windings) is placed in a B-field of magnitude .6 teslas going into the page (see Figure 19.6a). Although the magnetic field changes at a constant rate of -.067 tesla/second, it takes six seconds for the radius of the coil to increase to 30 centimeters and the orientation of the coil, relative to the B-field, to change as shown in Figure 19.6b. Assume all of the changes happen at a uniform pace, and accept my apology for dumping a lot of picky calculations on you (this wouldn't be the case on an AP test--I've done it this way to give you the chance to see lots of stuff you should be at least passingly aware of).
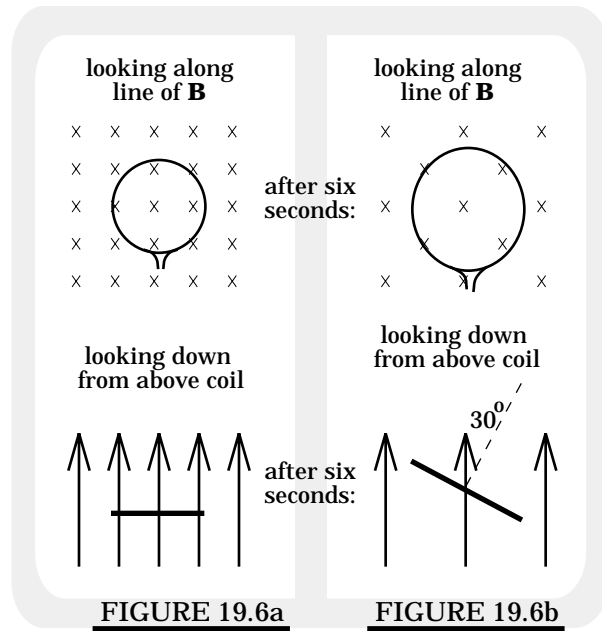
a.) Will this situation produce a constant induced EMF across the leads of the coil?

i.) To answer that, we need to determine what the EMF expression looks like. Using Faraday's Law and noticing that EVERYTHING is varying, we get:

$$\mathcal{E} = -N \ d(BA \cos \theta)/(dt)$$

$$= -N[A\cos\theta \ (dB/dt) + B\cos\theta \ (dA/dt) + AB(-\sin\theta)(d\theta/dt)].$$

ii.) What does this little gem tell us? It says that if each of the three parts identified in the above expression is constant over the 6 second period, the induced EMF will be a constant.

iii.) Unfortunately, none of the pieces are constant. (Not clear? Look at the first term. The dB/dt term is constant because we were told that all of the changes happened at a uniform pace, but the A term in $A\cos\theta$ (dB/dt) is a different value depending upon when you do the evaluation. So what A do you use? You have to do the evaluation for a particular point in time when A has a well defined value. Otherwise, you can't do the calculation).

looking along line of **B**

looking along line of **B**

after six seconds:

looking down from above coil

looking down from above coil

looking down from above coil

30°

after six seconds:

FIGURE 19.6a   FIGURE 19.6b

iv.) The bottom line is that we can determine what the induced EMF is at a given point in time during the 6 second period, assuming we can determine the angle at that point, but we can't determine a single numeric solution for the induced EMF during the whole 6 second period because the flux is not changing at a constant rate over that period (hence the EMF is not constant over that period).

b.) Given that we can't determine a constant EMF value for the whole 6 seconds, what is the EMF value at t = 3 seconds.

i.) Preliminary information:

1.) The rate at which the magnetic field changes is given as dB/dt = -.067 teslas/second.

2.) With constant change, the magnetic field at t = 3 seconds will be the initial magnetic field plus (3 sec)((dB/dt), or B(t=3) = .6 T + (3 s)(-.067 T/s) = .399 T.

3.) The initial area of the coil is $\pi R^2 = \pi(.2\ m)^2 = .126\ m^2$. The final area after 6 seconds of continuous uniform change calculates to .283 m^2. That means that the rate at which the area changes over that 6 second period will be dA/dt = (A_2 - A_1)/(t_2 - t_1) =(.283 m^2 - .126 m^2)/(6 sec) = .0262 m^2/sec.

4.) With constant change, the area at t = 3 seconds will be the initial area plus (3 sec)((dA/dt), or A(t=3) = .126 m^2 + (3 s)(.0262 m^2/s) = .2046 m^2.

5.) Noting that 30^0 is the same as .5326 radians, the rate of angular change (alias ω . . . remember, ω = dθ/dt?), measured in radians, is (.5326 rad)/(6 seconds) = .087 radians/second.

6.) The angle θ is defined as the angle between B and A. With uniform rotation, according to the sketch, that angle halfway through the 6 second rotation will be 15^0.

Note: You can use degrees here in Part 6 because you will use the angle ONLY inside a sine function . . . your calculator doesn't care what the angle's units are as long as it is set to take that particular measure. If you were using the angle in any other way (for instance, to determining ω), the angle would have to be written in radians.

ii.) Putting the whole mess together without including units (to save space), we get:

$\mathcal{E}$ = -N d(BA cos θ)/(dt)

= -N[A   cosθ (dB/dt) + B   cosθ (dA/dt) + A   B   (-sinθ) (dθ/dt)]
= -25[(.2046)(cos 15^0)(-.067)+(.399)(cos 15^0)(.0262)+(.2046)(.399)(-sin 15^0)(.087)]
= .125 volts.

c.) Finally, to complete the amusement, assuming the resistance in the coil's wire is 12 ohms, what is the induced current in the coil at t = 3 seconds?

i.) This is just Ohm's Law. Specifically,

$$\mathcal{E} = i\,R$$

$$\Rightarrow \quad i = \mathcal{E}\,/\,R$$

$$= (.125 \text{ volts})/(12\ \Omega)$$

$$= 1.04 \times 10^{-2} \text{ amps.}$$

C.) Direction of Induced Current--Lenz's Law:

    1.) So far, we have used Faraday's Law to determine the magnitude of the induced EMF and current in a coil through which there is a changing magnetic flux. What about current direction?
    Lenz's Law provides a way to determine the direction of an induced current. The approach is somewhat complex but certainly understandable if approached in an orderly manner. The best way to proceed is by looking at an example in pieces, then by putting the pieces together. Consider:

    a.) A coil is placed in a constant external magnetic field (see Figure 19.7).

    b.) As the external magnetic field passes through the coil's face, there is a magnetic flux through the coil.

    c.) At this point, there is no current in the coil as there is no changing flux through the coil.

FIGURE 19.7

    d.) Assume now that the external flux somehow increases through the coil.

Note: As was said above, this increase could come as a consequence of an increase in the magnitude of the external B-field, an increase in the area of the coil's face, a change in the angle between the area vector A and the magnetic field vector B, or any combination thereof.

    e.) As there is now a changing flux through the coil, there will be an induced EMF which, in turn, will cause an induced current in the coil. As we are not yet sure which way the current will flow, assume its direction is as shown in Figure 19.8a. If that be the case, the following will be true:
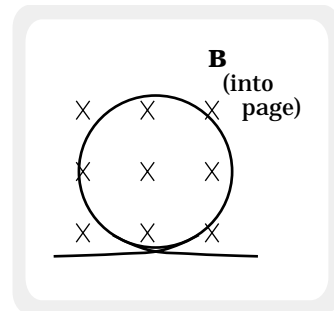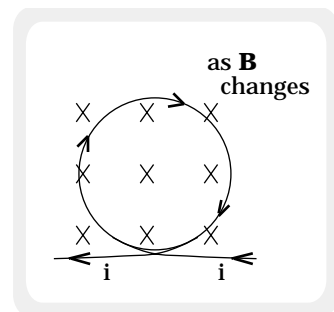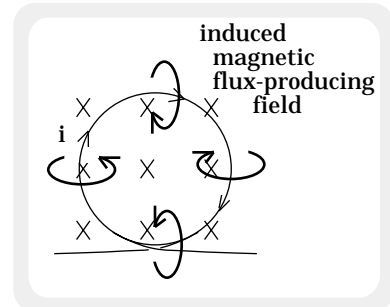

FIGURE 19.8a

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i.) The induced current will produce a magnetic field of its own (this will be referred to as the <u>induced</u> B-field).

ii.) This field will set up a magnetic flux through the coil's face (this will be referred to as the <u>induced</u> magnetic flux). Given the assumed current-direction as depicted in Figure 19.8b, that induced flux will ADD to the external flux through the coil's face (i.e., the net flux through the coil's face will be $\Phi_{m,external} + \Phi_{m,induced}$).

induced magnetic flux-producing field

<u>FIGURE 19.8b</u>

iii.) The logical consequences of this are as follows: As the induced flux adds to the increasing external flux, the net flux change becomes bigger than it would have been. As such, the induced current in the coil becomes bigger than it would have been which, in turn, creates an even bigger induced B-field through the coil. With the induced B-field larger than ex-pected, the induced flux becomes even GREATER, which means the net induced flux becomes GREATER, which creates an EVEN BIGGER current in the coil, etc., etc., etc.

iv.) <u>Bottom line #1</u>: If, as the external flux increases, the in-duced flux ADDS to it (i.e., helps the external flux out as it in-creases), the consequence will be a runaway situation in which the conservation of energy is wholly violated.

v.) <u>Bottom line #2</u>: The induced current must flow such that <u>its</u> magnetic flux does NOT aid the external flux change. In fact, the induced magnetic flux <u>must oppose that change</u>.

vi.) <u>Bottom line #3</u>: Why the negative sign in $\varepsilon = -N \ (d\Phi_m/dt)$? To denote that the EMF is opposing the flux change.

2.) Lenz's Law: When an EXTERNAL magnetic flux <u>CHANGES</u> through the face of a coil, an EMF is induced in the coil. That (induced) EMF generates an (induced) current which produces an (induced) magnetic field through the coil's face, which produces an (induced) magnetic flux through the coil's face, which is directed so as to OPPOSE the change of the <u>external</u> magnetic flux that started the whole process in the first place.

The right-thumb rule (thumb in direction of current--fingers curl in direction of B through the coil) allows you to determine the direction of the required induced current. (SEE NEXT SECTION NOW!)

3.) IMPORTANT--a SHORTCUT to using Lenz's Law (or, no matter how little the above made sense, the following will always work):

a.) If the external flux <u>INCREASES</u>, no matter how this is accomplished, the direction of the INDUCED B-field through the coil's face will always be <u>OPPOSITE</u> the direction of the external B-field. If the external flux <u>DECREASES</u>, the INDUCED B will be IN the direction of the external B-field.

b.) A CLEVER WAY TO GET THE INDUCED CURRENT'S DIRECTION--the "other" right-thumb rule: Determine the direction of the induced B-field using the logic outlined in Part 3a. Orient your <u>right thumb</u> along that line, pointing it through the coil. Your fingers will curl in the direction of the induced current in the loop.

4.) Using Lenz's Law, determine the direction of the induced current in the situations outlined below:

a.) The external B-field in Figure 19.9a is increasing out of the page, which means the external flux is increasing. The only way to oppose an increasing magnetic flux is to produce a second magnetic flux that subtracts from the original flux. In other words, we need a current that will produce a B-field in the direction opposite the existing B-field, or into the page.

front view

B (out of page) increasing

<u>FIGURE 19.9a</u>

Using the "other" right-thumb rule: Direct your right thumb into the page and through the coil. The fingers of your right hand curl clockwise. That's the direction of the induced current.
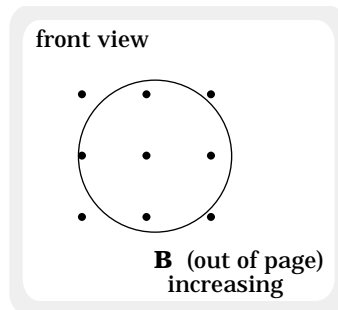
b.) The external B-field in Figure 19.9b is decreasing. The only way to oppose its decreasing magnetic flux is to produce a second magnetic flux that adds to the original flux. In other words, we need a current that will produce a B-field in the same direction as the external B-field, or to the right.

over head view

B decreasing

<u>FIGURE 19.9b</u>

Direct your right thumb to the right and through the coil (you'll obviously have to visualize doing this--the coil's face isn't actually shown). Your right-hand fingers will curl so that the induced current will flow toward the page's bottom in the wire section shown.
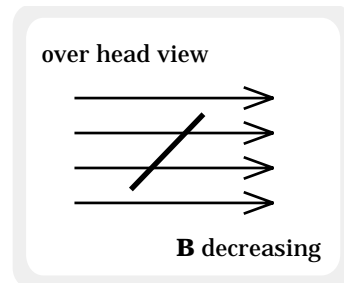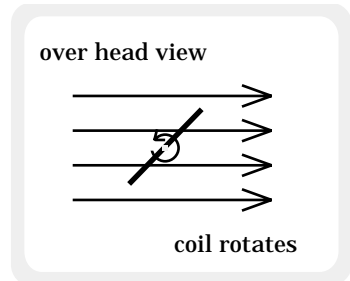
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c.) Rotation as shown in Figure 19.9c depicts a situation in which the external magnetic flux through the coil's face is increasing. The only way to oppose an increase in magnetic flux is to generate a magnetic flux that subtracts from the existing flux. In other words, we need a current that will produce a B-field in the direction opposite the external B-field, or to the left.

over head view

coil rotates

FIGURE 19.9c

Direct your right thumb to the left and through the coil (again, you'll have to visualize doing this as the coil's face isn't actually shown). Your right-hand fingers will curl so that the induced current will flow toward the top of the page in the wire section shown.

d.) Rotation as shown in Figure 19.9d depicts a situation in which the external magnetic flux through the coil's face is decreasing (the coil's magnetic flux is at a maximum going down). The only way to oppose a decrease in magnetic flux is to generate a magnetic flux that adds to the ex- isting flux. In other words, we need a current that will produce a B-field in the same direction as the external B-field, or into the page.

front view

rotating coil

pin

FIGURE 19.9d

Direct your right thumb into the page and through the coil. Your right-hand fingers will curl clockwise--that's the induced current's direction.
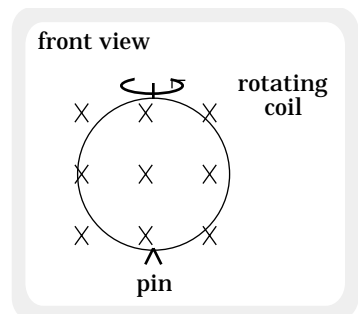
e.) As the area of the coil in Figure 19.9e becomes smaller, the magnetic flux through the coil's face decreases. The only way to oppose a de- crease in magnetic flux is to generate a magnetic field that adds to the existing field. In other words, we need a current that will produce a B- field in the same direction as the external B-field.

over head view

coil area diminishing

FIGURE 19.9e

Using the right-thumb rule: The current direction that generates such a magnetic field through the coil's face will be upward (toward the top of the page) in the section of wire visible to us in the sketch.
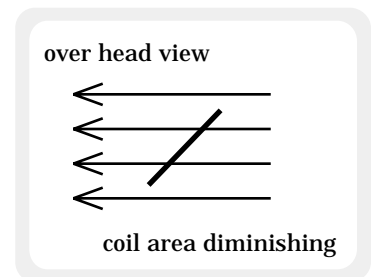
D.) Induced EMF's in Coils:

1.) The symbol for a coil in an electrical circuit is shown in Figure 19.10a. Coils are called a number of different things:

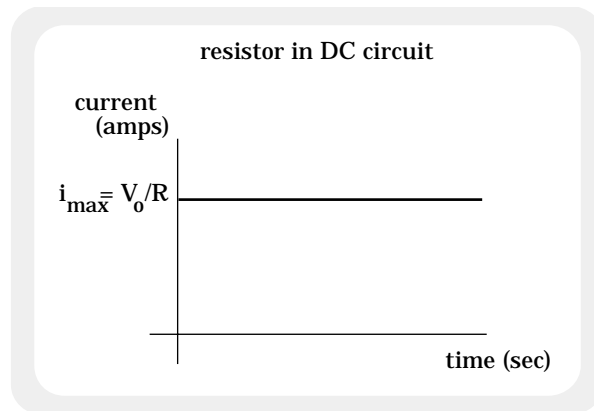circuit symbol for inductor

FIGURE 19.10a

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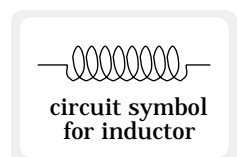
chokes, solenoids, or inductors. The last is the most common usage; the first is slang.

2.) Consider the coil, resistor, switch, and power supply in Figure 19.10b. At t = 0, the switch is closed. What will the Current versus Time graph look like for this situation?
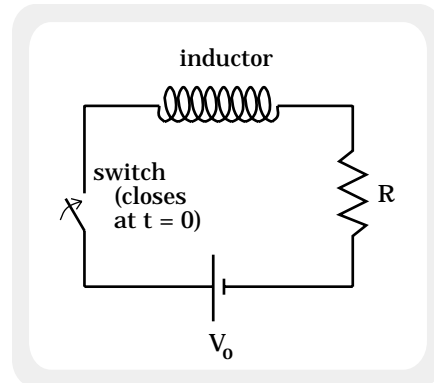
a.) If the circuit were comprised of nothing more than a resistor across the power supply, the voltage difference generated by the power source would create an electric field that would motivate all the free charge-carriers in the circuit to move at once. Current in the circuit would immediately be observed (the electric field sets itself up at just under the speed of light); its magnitude would be V_0/R; and its Current versus Time graph would be as shown in Figure 19.11a.

b.) With a coil in the circuit, everything changes.

FIGURE 19.10b

FIGURE 19.11a

i.) To begin with, we know that when current moves through a coil it creates a magnetic field down the coil's axis and a magnetic flux through the coil's face.

ii.) In the case cited above, there is no initial current in the coil, hence no initial magnetic field or magnetic flux. At t = 0, the switch is closed. As current begins to flow, the magnetic flux through the coil begins to INCREASE.

iii.) A changing magnetic flux induces an EMF (Faraday's Law) that attempts to set up B-field whose flux through the coil will oppose the external flux change that started the whole process.

iv.) Put another way, the sudden increase in current elicits a back-EMF that attempts to diminish the current increase through the coil. This opposition is not so large that it completely stops current build-up; it simply slows it. As such, the current in the

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circuit increases more slowly than would have been expected and the Current versus Time graph ends up looking like that shown in Figure 19.11b.

inductor and resistor in DC circuit

current (amps)

$i_{max} = V_o/R_{net}$

time (sec)
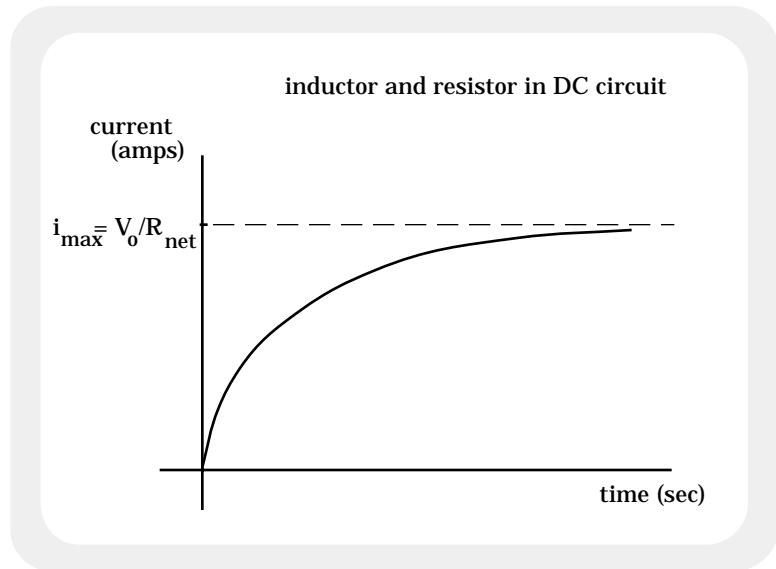
FIGURE 19.11b

Note 1: The current will sooner or later reach a maximum steady-state value. At that point, there will be no voltage drop across the coil <u>except</u> that generated by the resistance inherent within the coil's wire. As such, the current is governed by Ohm's Law and equals V_o/R_{net}.

Note 2: A similar situation exists when the switch is opened and the current tries to shut off. The coil will setup an EMF that will oppose the change--an EMF that attempts to keep current flowing in the circuit. Circuits with inductors in them do not turn off immediately.

E.) Inductance:

1.) So far, we have hand-waved through most of our discussion of EMF's in coils. It is time to examine the math.

2.) According to Faraday's Law, when the current through a coil changes, a back-EMF is produced that opposes the change of flux. That EMF can be calculated using the relationship:

$$EMF = -N \ (\Delta\Phi_m)/(\Delta t).$$

The problem with the above quoted expression is convenience--flux is not a quantity we can easily measure with normal laboratory equipment.

3.) Instead of linking the EMF to the change of flux, we could link it to the change of current that causes the change of flux. That is, we could define a proportionality constant L that allows us to relate the induced EMF across the coil with the change of <u>current</u> in the coil.

Mathematically, this would look like:

$$EMF = -L\frac{di}{dt}.$$

4.) The proportionality constant L is called the coil's inductance (you can now see why a coil in an electrical circuit is usually called an inductor). Its units are henrys, and it is not uncommon to find inductors in the milli-henry range (a milli-henry is 10^{-3} H and is symbolized as mH).

Qualitatively, inductance tells us how large an induced EMF (in volts) we can expect across the coils of an inductor per change of current per unit time.

Note: You will not be tested directly on the derivations below in Part 5. The section has been included because the math is interesting and is typical of the type of work done in mathematical physics (this is actually a sub-discipline in physics). Skim through it to get an idea of how the math works, then focus on the end result. Understanding the bottom line is important.

5.) Consider an electrical circuit in which there is an inductor whose inductance is L, a resistor whose resistance is R, a power supply, and a switch (for now, we will assume that any resistance wrapped up in the wires making up the coil is negligible). If the switch is closed at t = 0 seconds, what will the current in the circuit do as time progresses? (Note that this is the situation we graphed in the previous section--Figure 19.10b shows the circuit).

a.) According to Kirchoff's Laws, we can write a loop equation for the circuit. Doing so yields:

$$-L\frac{di}{dt} - iR + V_o = 0$$

$$\Rightarrow \quad \frac{di}{dt} + i\left(\frac{R}{L}\right) = \frac{V_o}{L}.$$

b.) This is a differential equation. It essentially states that we are looking for a function i such that when you take its derivative di/dt and add to it a constant times itself (i.e., (R/L)i), we always get the same number (in this case, V_0/L).

c.) From experience, the solution to a differential equation of this form is:

$$i = A + Be^{kt},$$

where A, B, and k are all constants to be determined.

d.) Solving for the constants is essentially a boundary value problem. That is, we must use what we know about the system at its boundaries--at $t = 0$ and at $t = \infty$. Doing so yields:

i.) At $t = 0$, the current i through the circuit will equal ZERO (see the graph). Using that, we can write:

$$i = A + Be^{kt}$$
$$\Rightarrow \quad 0 = A + Be^{k(0)}$$
$$\Rightarrow \quad B = -A.$$

ii.) At $t = \infty$, the current i in the circuit will equal $i_0 = V_0/R$. Using that and the information gleaned above, we can write:

$$i = A - Ae^{kt}$$
$$\Rightarrow \quad i_0 = A - Ae^{k(\infty)}.$$

The only way this can not be infinitely large is if the constant k is negative. From physical constraints (i.e., the fact that there is a limit on the amount of current the circuit can have), we can un-embed the negative sign inside k and rewrite the above equation as:

$$i_0 = A - Ae^{-k(\infty)}$$
$$= A - A\left(\frac{1}{e^{k(\infty)}}\right)$$
$$\Rightarrow \quad i_0 = A.$$

iii.) Using the information gleaned from above, we can write the current as:

$$i = i_0 - i_0 e^{-kt}.$$

The derivative di/dt of this function is:

$$di/dt = i_0 k e^{-kt}.$$

iv.) Going back to Kirchoff's loop equation, we can write:

$$-L\frac{di}{dt} - iR + V_o = 0$$

$$\Rightarrow \quad -L\left(i_o k e^{-kt}\right) - \left(i_o - i_o e^{-kt}\right)R + V_o = 0$$

$$\Rightarrow \quad -L\left(i_o k e^{-kt}\right) + \left(R i_o e^{-kt}\right) - i_o R + V_o = 0.$$

v.) The only way this expression can be satisfied is if the time-dependent exponential terms add to zero and the non-exponential constants add to zero. Following this line of reason, we can write:

$$i_o = V_o/R$$

and

$$-L i_o k e^{-kt} + R i_o e^{-kt} = 0.$$

vi.) After canceling the current and exponential terms, this last expression leaves us with:

$$k = R/L.$$

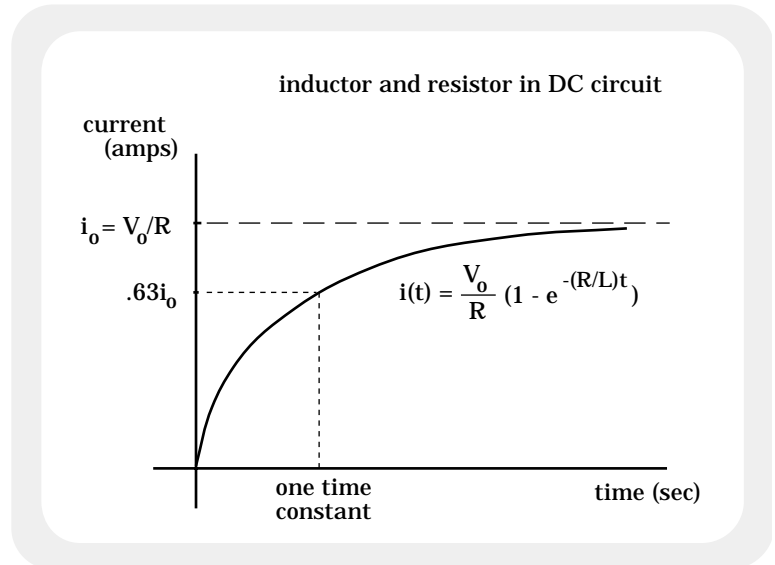e.) Putting it all together, we get:

$$i = i_o(1 - e^{-(R/L)t}).$$

6.) The time dependent expression for current in an RL circuit is similar to the charging function in an RC circuit. As was the case with capacitors, we can define a time constant τ_L for our circuit. As before, one time constant will equal the amount of time needed for $e^{-Rt/L}$'s <u>exponent</u> to numerically equal -1. In our case, this occurs when $t = L/R$.

a.) The consequences of this can be seen by doing the math:

$$i(t) = i_o\left(1 - e^{-Rt/L}\right)$$

$$\Rightarrow \quad i\left(t = \frac{L}{R}\right) = i_o\left(1 - e^{-R\left(\frac{L}{R}\right)/L}\right)$$

$$= i_o(1 - .37)$$

$$= .63 i_o.$$

b.) Knowing the time constant allows us to determine how fast the current will rise or fall when a DC-driven RL circuit is turned on or off. According to the math, after one time constant the current will be 63% of its maximum (after two time constants, the current will be 87% of its maximum).

c.) A graph of the current function and the time constant is shown in Figure 19.11c. Notice that it is exactly the graph we deduced using hand-waving arguments in previous sections.

inductor and resistor in DC circuit

current (amps)

$i_o = V_o/R$

$.63i_o$

$i(t) = \dfrac{V_o}{R} (1 - e^{-(R/L)t})$

one time constant

time (sec)

FIGURE 19.11c

F.) Energy Stored in an Inductor:

1.) To increase the current in a coil, extra work must be done to overpower the coil's tendency to resist changes in its magnetic flux. Where does that energy go? Some of it is stored in the inductor's magnetic field. This section deals with how much energy a current-carrying coil can store.

2.) To determine the amount of energy wrapped up in an inductor's magnetic field:

a.) Reconsider Kirchoff's loop equation. That is:

$$L\frac{di}{dt} + iR = V_o.$$

Multiplying by i, we get:

$$iL\frac{di}{dt} + i(iR) = iV_o.$$

b.) Noting that the iV_o term equals the amount of power provided to the circuit by the power supply, and the i^2R term equals the amount of power dissipated by the resistor, it's a good bet that the Li(di/dt) term equals the amount of power dissipated by the inductor in the circuit.

The resistor pulls energy out of the circuit as heat. The inductor pulls energy out of the system by storing it in the coil's magnetic field.

As work per unit time (i.e., the power) equals dW/dt, we can write:

$$\frac{dW}{dt} = Li\frac{di}{dt},$$

or

$$dW = Li(di).$$

Assuming we want to sum all the work done on the inductor in storing energy from the time the current is zero to the time it is at some maximum current i_o, we can integrate both sides to determine the total energy stored by the inductor. Doing so yields:

$$energy = \int dW$$
$$= \int_{i=0}^{i_o} Li(di)$$
$$= \frac{1}{2}Li_o^{\,2}.$$

Note: The energy in an inductor is stored in a magnetic field governed by current flow in the coil, and the energy expression is $(1/2)Li^2$. The energy in a capacitor is stored in an electric field governed by a voltage across the capacitor's plates, and the energy expression is $(1/2)CV^2$. Nice symmetry.

G.) Transformers:

1.) Consider the iron yoke (the doughnut-shaped structure) in Figure 19.12a.

a.) The primary circuit is comprised of a switch and power supply connected to a coil whose winds are wrapped around the yoke. The secondary circuit is comprised of a galvanometer connected to a secondary coil the winds of which are also wrapped around the yoke.
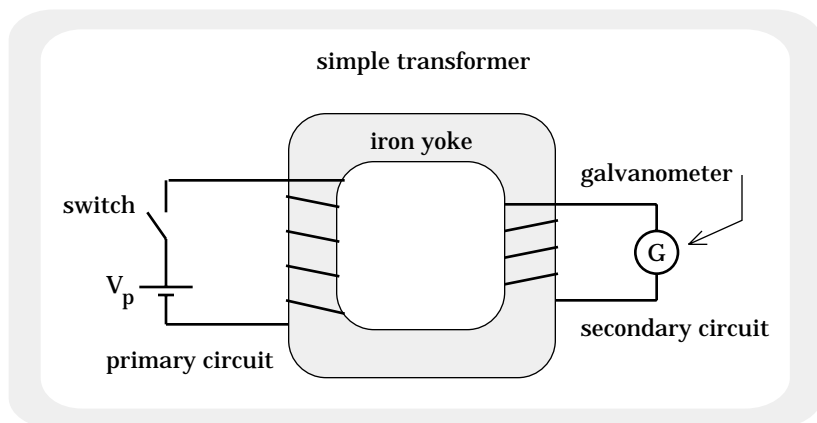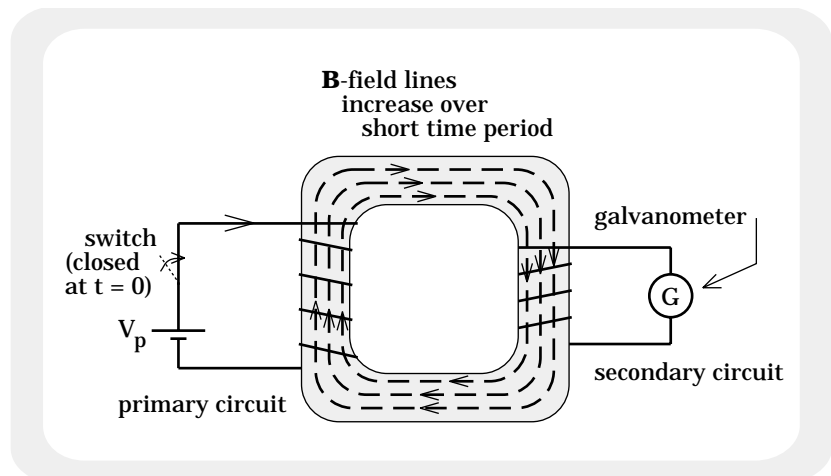

FIGURE 19.12a

b.) At t = 0, the switch is closed. Current establishes itself in the primary circuit, building slowly due to the presence of the coil.

Note: Why does the current increase slowly? You should know from the last section! If you do not, read on:

Initially there is no magnetic flux through the primary coil as there is no current (the switch is initially open). When the switch is closed, the current in the primary coil begins to increase. In doing so, a magnetic field down the primary coil's axis appears which produces a magnetic flux through the primary coil's face. According to Faraday's Law, this generates an induced EMF in the primary coil which opposes the increasing flux through the primary coil. As the flux-producing magnetic field is generated by the current in the primary circuit, this back-EMF effectively fights that current. That is why the primary circuit's current grows slowly.

c.) The yoke is made of iron. As the current increases in the primary circuit, the primary coil's magnetic field magnetizes the yoke (see Figure 19.12b). Being one piece, this magnetic field sets itself up throughout the yoke and passes through the face of the secondary coil.

FIGURE 19.12b

d.) Because the magnetic field through the secondary coil is changing, an induced EMF will be set up in the secondary coil to oppose the changing flux. As such, current will flow in the secondary circuit. THIS CURRENT FLOW WILL CONTINUE ONLY AS LONG AS THE MAGNETIC FLUX CHANGES IN THE SECONDARY COIL. During that time, the galvanometer will register charge flow.

e.) Once the current in the primary coil has built to steady-state, there will be a magnetic field in the yoke and the secondary coil, but there will not be a CHANGING magnetic field. As such, there will be no induced EMF in the secondary coil and the galvanometer will read no current in that part of the system.

f.) If, after a time, the switch in the primary circuit is opened, the exact opposite scenario will occur. The current in the primary circuit will decrease relatively slowly, decreasing the magnetic field through the primary coil. The diminishing magnetic field through the primary coil will diminish the magnetic field setup in the yoke which, in turn, will diminish the magnetic flux through the secondary coil. The secondary coil will respond by producing an induced EMF to oppose the change of flux through its face, generating a current in the secondary circuit for as long as the changing magnetic field exists.

A graph of the current in the secondary circuit as a function of time is shown in Figure 19.12c.

induced current in secondary coil

switch closes producing brief current spike

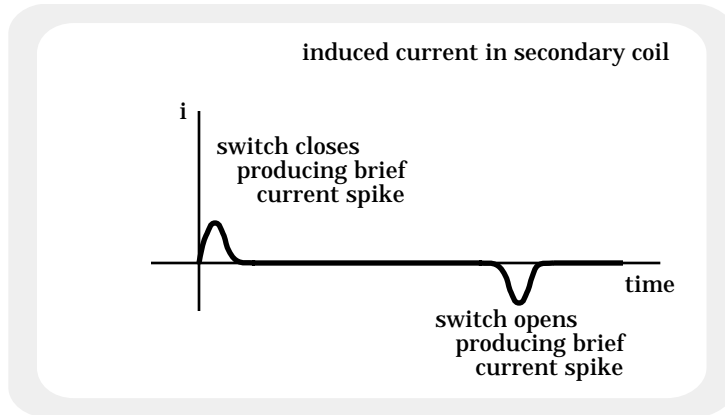switch opens producing brief current spike

FIGURE 19.12c

2.) This device is called a transformer. It allows us to transfer power from one electrical circuit (the primary) to another electrical circuit (the secondary) without electrically connecting the two. It is not particularly useful in DC circuits where the only change in current occurs when a switch is opened or closed, but it is very useful in AC circuits.

H.) Transformers and AC Circuits:

1.) There is another kind of power source we have yet to discuss--the alternating current (AC) power supply.

a.) In a DC circuit, the power source supplies a terminal voltage which sets up a constant electric field. Free charge carriers in the circuit all respond to that field by moving in one direction only.

b.) In an AC circuit, the power source supplies an alternating terminal voltage that sets up an electric field that changes both in magnitude and direction. Free charge carriers in an AC circuit respond to this alternating field by jiggling back and forth.

2.) With this in mind, consider what happens when an AC power supply is put in place of the DC power supply originally used in the transformer's primary circuit:

a.) As stated above, the current in the primary circuit will be constantly changing in direction and magnitude.

b.) A constantly changing, alternating current will produce a constantly changing, alternating magnetic field in the yoke. This, in turn, will produce a constantly changing, alternating magnetic flux through the secondary coil.

c.) An alternating magnetic flux through the secondary coil will produce an alternating induced EMF across the secondary coil's terminals which, in turn, will produce an induced alternating current in the secondary circuit.

d.) Bottom line: A transformer is useful if one wants to transfer AC power from one circuit to another without electrically hooking the two circuits together. As the current in the secondary coil is constantly varying, power in such circuits is transferred continuously.

Note: Putting an AC power source in place of the DC source in Figure 19.12a will additionally require changing the current-measuring device in the secondary circuit. Galvanometers are DC ammeters; we would need an AC ammeter.

I.) The Mathematics of Transformers:

NOTE: More will be said about the mathematics of transformers in the context of AC circuits in the next chapter. For now, some general mathematical observations can be made.

1.) Assuming $\mathcal{E}_p$ is the induced EMF across the primary circuit at a given instant, i_p is the induced current through the primary circuit at that same instant, etc., and assuming 100% efficiency (that is, assuming all the power provided by the magnetic component of the primary coil is transferred to the secondary coil), we can write:

$$P_p = P_s$$
$$i_p \mathcal{E}_p = i_s \mathcal{E}_s.$$

2.) We know that $\mathcal{E} = -N \, (\Delta\Phi_m)/(\Delta t)$, and we know that the change in flux $\Delta\Phi_m/\Delta t$ through the primary and secondary coils will be the same (both coils have the same face-area and the same magnetic field passing through them). With this information we can write:

$$i_p \mathcal{E}_p = i_s \mathcal{E}_s$$
$$i_p \, [-N_p(\Delta\Phi)/(\Delta t)] = i_s \, [-N_s(\Delta\Phi)/(\Delta t)].$$

Canceling out the $\Delta\Phi/\Delta t$ variables on both sides of the equal sign and manipulating, we get:

$$N_p / N_s = i_s / i_p.$$

3.) Important observation: The turns-ratio N_p/N_s is inversely proportional to the ratio of currents in the primary and secondary coils.

4.) Going back to the power relationship, we know that:

$$i_p \mathcal{E}_p = i_s \mathcal{E}_s$$
$$\Rightarrow \quad i_s/i_p = \mathcal{E}_p / \mathcal{E}_s.$$

Substituting this into our turns relationship

$$i_s / i_p = N_p / N_s$$

yields

$$N_p / N_s = \mathcal{E}_p / \mathcal{E}_s.$$

5.) Important Observation: The turns-ratio is directly proportional to the voltage ratio between the primary and secondary coils.

6.) <u>Summary</u>:

   a.) When $N_p > N_s$:

$$\mathcal{E}_p > \mathcal{E}_s \text{ and } i_p < i_s.$$

This is called a <u>step-down transformer</u> because the voltage steps down (i.e., decreases) as we go from the primary to the secondary coil. Notice that with a step-down transformer, the <u>current</u> in the secondary coil is GREATER THAN the current in the primary.

The turns-ratio is still such that:

$$N_p/N_s = \mathcal{E}_p/\mathcal{E}_s \quad \text{and} \quad N_p/N_s = i_s/i_p.$$

b.) When $N_p < N_s$:

$$\mathcal{E}_p < \mathcal{E}_s \quad \text{and} \quad i_p > i_s.$$

This is called a <u>step-up transformer</u> because the voltage steps up (i.e., increases) as we go from the primary to the secondary coils. Notice that with a step-up transformer, the <u>current</u> in the secondary coil is LESS THAN the current in the primary.

Technical Note: The symbol for a transformer in a circuit is shown in Figure 19.12d. The symbol is supposed to represent two coils connected by a common magnetic flux.
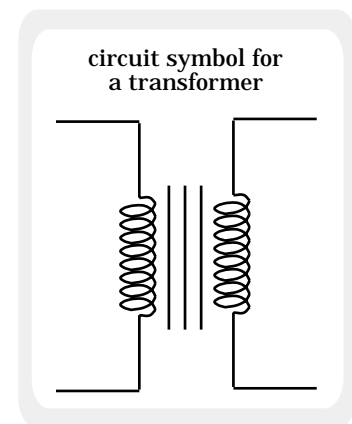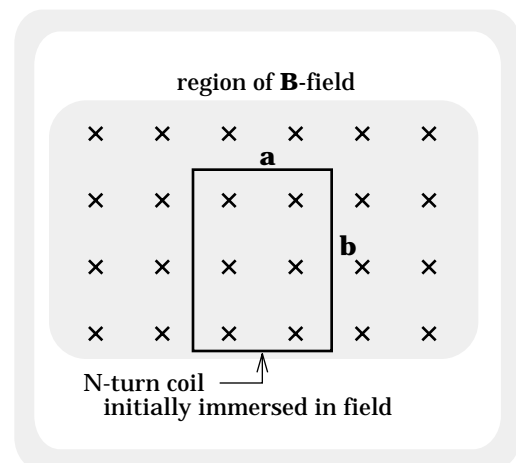
J.) Motional EMF's:

1.) An induced EMF will be generated whenever a coil undergoes any kind of changing flux through its face. If that changing flux is generated by moving the coil either into or out of a magnetic field, the resulting EMF is called a motional EMF.

2.) Example: An N wind rectangular coil whose dimensions are a by b (see Figure 19.13a) is pulled with constant velocity v out of a known, constant magnetic field B. What is the induced EMF in the wire?

a.) Begin by determining a general expression for the magnetic flux through the coil at an arbitrary

circuit symbol for a transformer

FIGURE 19.12d

region of **B**-field

N-turn coil initially immersed in field

FIGURE 19.13a

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point in time. From Figure 19.13b, that will be:

$$\Phi_m = B \bullet dS$$
$$= B[a(b-x)]\cos 0^\circ$$
$$= Bab - Bax.$$

Important Note: The AREA term in the net-flux expression is not necessarily the area of the coil-- it is the <u>area through which the magnetic field passes</u>.

coil pulled out of field
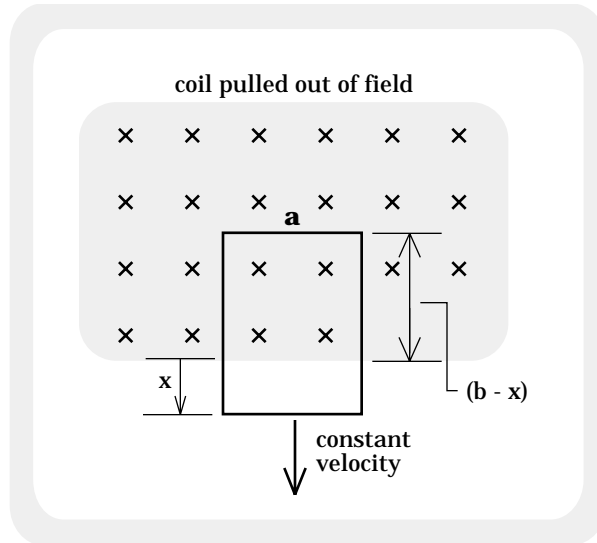
FIGURE 19.13b

b.) Knowing the magnetic flux, we can use Faraday's Law to write:

$$\varepsilon = -N\frac{d\Phi_m}{dt}$$
$$= -N\frac{d(Bab - Bax)}{dt}$$
$$= NBa\frac{dx}{dt}$$
$$= NBav.$$

3.) Additional follow-up questions:

a.) Using Lenz's Law, in which direction does the current flow?

<u>Solution</u>: The flux is decreasing through the coil's face, so the induced current must produce an induced flux that will <u>oppose</u> the change by adding to the decreasing external flux. This will require a B-field into the page (i.e., in the same direction as the external B-field), which will be accomplished with an induced clockwise current.

b.) What is the current in the wire if the net resistance in the wire is R?

i.) Using Ohm's Law, we get:

$$\mathcal{E} = iR$$

$$\Rightarrow \quad i = \mathcal{E}/R$$
$$= NBav/R.$$

c.) Neglecting gravity and assuming the velocity v is constant (i.e., acceleration is zero), what is the force applied to the coil as it is pulled out of the magnetic field?

i.) A current-carrying wire in a magnetic field feels a magnetic force equal to F = iLxB. At a given instant, there are four sections of wire within our rectangle, each supporting the same amount of induced current but each with charge-flow in a different direction. The net force on the loop as a whole will be the vector sum of the four forces generated by this current flow.

To make things clear, each section of wire has been labeled in Figure 19.14 and will be initially treated as a separate entity:
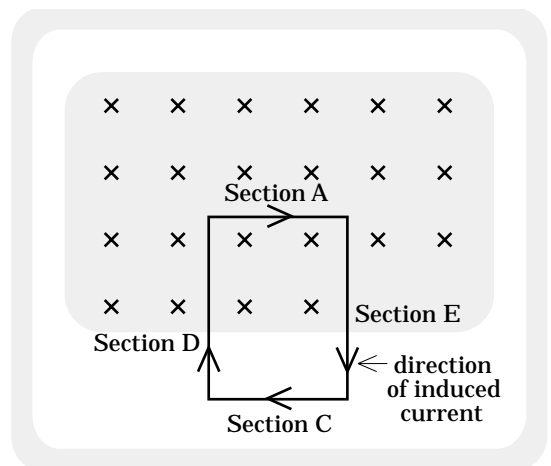
FIGURE 19.14

ii.) WIRE SECTION A: Using the right-hand rule, the direction of the force applied to wire Section A due the interaction of the magnetic field and the current in that section of wire will be upward toward the top of the page.
The magnitude of the magnetic force will be:

$$F_A = iLB \sin 90^o$$
$$= iaB.$$

Substituting in for the current found in Part b above, we get:

$$F_A = iaB$$
$$= (NBav/R) \, aB$$
$$= Na^2B^2v/R.$$

iii.) WIRE SECTION E: Although the most likely next step would be to determine the magnitude of the force on wire E, a little judicious observation will come in handy here. Notice that the magnitude of the force on wire-sections E and D are the same (they

both have the same length, current, and are in the same magnetic field). Additionally, the direction of the force on Section E is to the right while the direction of the force on Section D is to the left. These two equal and opposite forces add to zero, leaving us with no reason to continue dealing with them.

   iv.) WIRE SECTION C: As Section C is not in the magnetic field, it will experience no magnetic force.

   v.) Using Newton's Second Law in the only direction in which there is a non-zero magnetic force (i.e., in the y direction), and remembering there must be an applied force (F_{you}) to counteract that magnetic force if the wire is to move with a constant velocity, we get:

$$\underline{\Sigma F_y}:$$
$$-F_{you} + Na^2B^2v/R = ma_y = 0$$
$$\Rightarrow \quad F_{you} = Na^2B^2v/R.$$

Note: It does not matter whether the coil is moving into or out of a magnetic field, the CHANGING FLUX WILL ALWAYS GENERATE A CURRENT THAT INTERACTS WITH THE EXTERNAL MAGNETIC FIELD TO PRODUCE A MAGNETIC FORCE THAT <u>OPPOSES THE MOTION</u>. That is, if the coil is being pushed INTO the field, the induced force will try to keep it out; if the coil is being pulled OUT OF the induced field, the force will try to keep it in. Induced forces always FIGHT THE CHANGE.

K.) Eddy Currents . . . Briefly:

   1.) We now know what happens to a coil that moves into or out of a magnetic field. An induced magnetic force on the coil opposes the motion.

     a.) Whether a material is magnetic or not has nothing to do with this phenomenon. A copper wire is not magnetic, yet we saw the effect on a copper-wire loop in the above example. The key is in the fact that every metallically bonded structure has charge carriers that can move freely within the structure.

   2.) Consider: An aluminum plate (note--the plate is metallic but is not magnetizable) rotates counterclockwise about its central axis. At a given instant, a fixed magnetic field that is oriented out of the page and that pierces a small portion of the plate is turned on.

a.) Although there are no defined current paths as would be the case with a wire loop, we can define any number of imaginary loop-paths on the plate's face. Two such paths (A and C) are shown in Figure 19.15.

spinning aluminum plate

an imaginary Loop A entering **B**-field

eddy current (clockwise)

B-field (out of page)

counterclockwise plate rotation

eddy current (counterclockwise)

an imaginary Loop C leaving **B**-field

FIGURE 19.15

b.) For now, ignore everything except Loop A on the sketch. As the plate spins, Loop A sooner or later enters the magnetic field. As it does, a magnetic flux through its face begins to increase and a clockwise current (Lenz's Law) is induced. This current is called an eddy current.

c.) The part of the eddy current that is in the magnetic field feels a force due to its presence in that field (remember iLxB?). The direction of that force, as determined by the cross product, is opposite the direction of the physical motion of the current path as it <u>enters</u> the B-field.

d.) As Loop A leaves the magnetic field (i.e., when it gets to the position shown as Loop C), the magnetic flux decreases producing a counterclockwise eddy current. The part of the eddy current that is in the magnetic field at this point feels a force due to its presence in that field. The direction of that force is, again, opposite the direction of the physical motion of the current path as it <u>leaves</u> the B-field.

e.) In both cases the eddy current's interaction with the magnetic field slows the rotation.

f.) Although a loop will generate a force only when entering or leaving the magnetic field (i.e., only when the magnetic flux through the loop's face is changing), there will always be some loop making its entrance while another is making its exit. The eddy currents setup in these loops will always interact with the fixed magnetic field to produce a retarding force that acts to slow the plate's motion.

g.) This device is an eddy current brake. It will work on any whole, metallic material whether it be magnetizable or not.
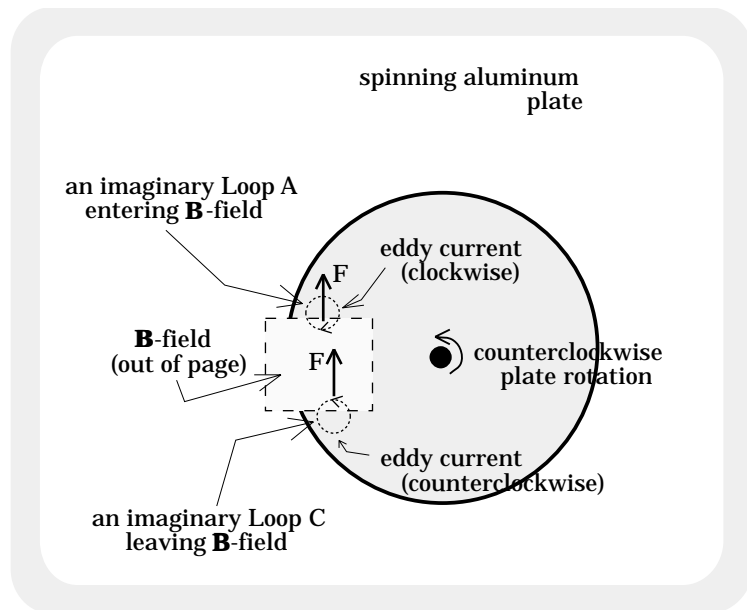
L.) The Electric Field Generated by a Changing Magnetic Flux:

1.) We know that a voltage difference across the terminals of a power supply will produce an electric field in the wires of an electrical circuit. It is that electric field that motivates charge carriers in the wires to move. Though it was not mentioned in the Circuits chapter, a power supply's electric field is generated by static charge at the terminals.

The relationship between an electrical potential difference and the electric field it generates was first examined while considering static charge situations. The question arises, "Do electric fields and electrical potential differences behave in the same way when a circuit's power comes from a changing magnetic flux, versus from an EMF provided by a battery?"

2.) Let's begin by re-examining the static situation. In a nutshell, our theory suggests that:

a.) When dealing with a constant electric field E, the voltage difference ΔV between two points in the field is:

$$\Delta V = -\mathbf{E} \cdot \mathbf{d} ,$$

where d is a displacement vector between the first and second point.

Note: What is actually being done here? The $\mathbf{E} \cdot \mathbf{d}$ term calculates how much work per unit charge is available between the two points. By definition, negative that amount equals the electrical potential difference between the two points.

b.) If the electric field is variable or the path non-linear, the work per unit charge must be determined for motion over an arbitrary, differential length dl along the path, then the total work is determined using integration. That is:

$$\Delta V = -\int \mathbf{E} \bullet \mathbf{dl},$$

where E is the electric field evaluated along the differential section and dl is a differential length-vector over that section in the direction of current.

3.) Executing this mathematical operation on a normal, DC electrical circuit (see Figure 19.16), noting that the constant electric field proceeds from the high voltage terminal to the low voltage terminal, that the

DC circuit

r

direction of **E**

+ −

power supply
(voltage V_0)
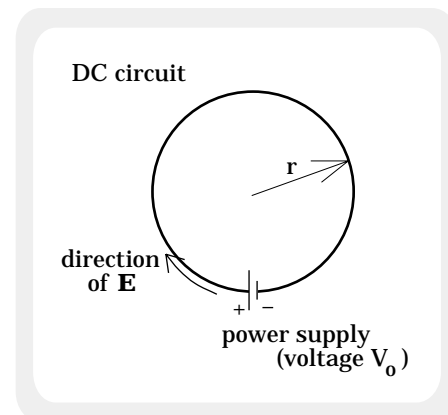
FIGURE 19.16

256

EMF in the circuit is the voltage difference across the terminal (i.e., V_0), and that the electric field goes almost around the entire circle (we can ignore the small amount not included due to the terminal connections), we can write:
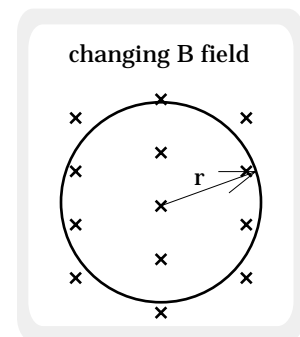
$$\Delta V = -\oint E \cdot dl$$
$$\Rightarrow \quad \left(V_{final} - V_{initial}\right) = -\oint E(dl) \cos 0^{\circ}$$
$$\Rightarrow \quad \left(0 - V_o\right) = -E \oint (dl)$$
$$\Rightarrow \quad V_o = E(2\pi r)$$
$$\Rightarrow \quad E = \frac{V_o}{2\pi r}.$$

Note 1: If we hadn't been careful about which voltage was initial and which was final, we could have run into sign problems. Assuming that we are traversing in the direction of the electric field so the angle between E and dl is zero, the voltage change around the path will be $-V_0$ (this is $-\mathcal{E}$ if expressed as an EMF).

Note 2: An integral sign with a circle on it denotes a line integration around a closed path. We have assumed the path is nearly closed, so to a good approximation the symbol is legitimate.

4.) Having examined the static situation, consider a loop in a time-varying magnetic field:

a.) A single loop of wire sits in a changing magnetic field (see Figure 19.17). According to Faraday's Law, the changing magnetic field creates a changing flux through the coil's face which, in turn, creates an induced EMF. The EMF, whose units are volts (this is true whether generated by a battery or by a changing magnetic field), creates an electric field which motivates charge in the loop to move (i.e., we get a current).

changing B field

FIGURE 19.17

b.) If our static charge model holds for changing magnetic fields, we should be able to determine the magnitude of that electric field. Using the approach, we get:

$$\Delta V = -\oint E \bullet dl$$

$$= -\oint E(dl) \cos 0°$$

$$= -E \oint (dl)$$

$$= -E(2\pi r).$$

c.) Unfortunately, there is an enormous problem here. Because the loop is connected to itself (unlike a loop broken by the DC power supply used in the previous problem), the voltage difference ΔV around the closed path should be zero (the integration comes back to its starting point, so there should be no voltage change around that path). But clearly, the right-hand side of the integral is not zero. So what is happening?

d.) By definition, an electric field is a modified force field. When generated by a static charge, that force field is additionally a conservative one. As a conservative force field can have a potential energy function associated with it, we can define an electrical potential function (i.e., $V = U/q$) for our field.

Although it isn't obvious at first glance, an electric field generated by a changing magnetic flux is not associated with a conservative force. As such, it is nonsense to speak of an electrical potential function for such a case because there is no chance the force field generated in the situation can ever have a potential energy function associated with it.

5.) Conclusion: The electric field set up by an induced EMF is related to the EMF, but not as would have been the case if the EMF had been generated by static charge. Specifically:

a.) If we let $\Delta V = -V_{bat} = -\mathcal{E}$, then E and the EMF are related as:

$$-EMF = -\oint \mathbf{E} \bullet d\mathbf{l}.$$

b.) This expression, coupled with Faraday's Law, yields:

$$N \frac{d\Phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l}.$$

c.) In the case of an electrical circuit where there is only one loop, N is 1 and our expression becomes:

$$\frac{d\Phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l}.$$

d.) <u>Bottom line 1</u>: When dealing with an electric field produced by a static charge (including fields found in standard electrical circuits with battery-produced EMF's), we can write:

$$\Delta V = -\int \mathbf{E} \bullet d\mathbf{l}.$$

e.) <u>Bottom line 2</u>: When dealing with an electric field produced by changing magnetic fields and induced EMF's, we must write:

$$\frac{d\Phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l}.$$

6.) Example: If the magnetic field in our circuit (see Figure 19.17) varies as k/t, determine the induced electric field E(t) in the wire:

$$\frac{d\Phi_m}{dt} = -\oint E \bullet dl$$

$$\Rightarrow \quad \frac{d(B \bullet A)}{dt} = -\oint E(dl) \cos 0^\circ$$

$$\Rightarrow \quad \frac{d\left(\left[\frac{k}{t}\right]\left[\pi r^2\right]\right)}{dt} = -E\oint (dl)$$

$$\Rightarrow \quad (k\pi r^2)\frac{d\left[\frac{1}{t}\right]}{dt} = -E(2\pi r)$$

$$\Rightarrow \quad (k\pi r^2)\left[-\frac{1}{t^2}\right] = -E(2\pi r)$$

$$\Rightarrow \quad E = \left[\frac{kr}{2t^2}\right].$$

QUESTIONS

19.1) Consider the N-turn coil shown in Figure I. It is made to rotate at a constant angular speed ω about the axis shown.

a.) At what angle ξ (see sketch) will the magnetic flux through the coil always be zero?

b.) Assume $\xi = 30^0$, the coil radius r = .15 meters, N = 120 turns, and B = .065 teslas. What is the maximum magnetic flux through the coil?

FIGURE I
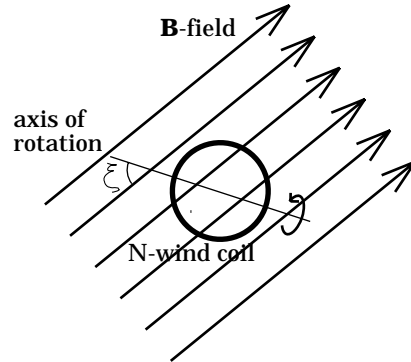
19.2) Each of the loops in Figure II are identical. Each has a length of .2 meters, a width of .08 meters, and a resistance of 4 ohms. Each is moving with a velocity magnitude of .28 m/s, and Loops A, C, and F each have .05 meters of their lengths not in the magnetic field at the time shown in the sketch (that is, the length outside the field at the time shown is .05 meters for each of those loops). The magnetic field in the shaded region is into the page with a magnitude of B = 3×10^{-2} teslas.

a.) What is the direction of the induced current for each loop at the instant shown in the sketch?

b.) What is the induced EMF generated in Loops A, C, and F at the instant shown?

c.) What is the magnitude and direction of the induced magnetic force felt by Loop F at the instant shown?

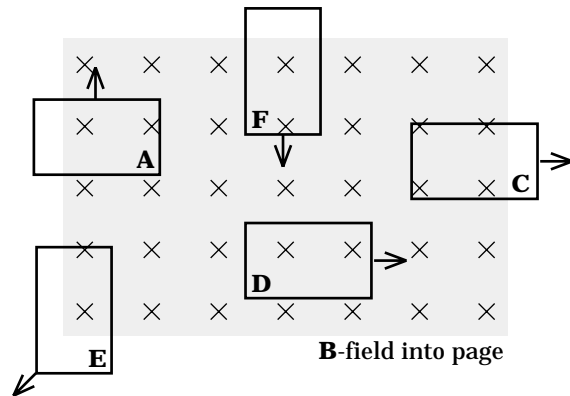d.) What is the direction of the induced magnetic force on Loops A, C, and D at the instant shown?

FIGURE II

19.3) A 6-turn circular coil whose radius is .03 meters and whose net resistance is 12 Ω's is placed squarely (that is, A and B are parallel to one

another) in a magnetic field whose direction is out of the page and whose magnitude is 2.3 teslas.

a.) What is the coil's initial magnetic flux?

b.) If the field increases at a rate of .6 teslas per second, what is the magnitude and direction of the induced current in the coil?

c.) Go back to the original situation. The coil is made to rotate about its vertical axis at an angular frequency of $\omega = 55$ radians per second. That means the induced EMF is AC.

i.) What is the frequency of the AC current generated?

ii.) Determine an expression for the induced EMF in the circuit.

19.4) An 80-turn square loop has side lengths of .12 meters. It is put squarely in a magnetic field whose direction is into the page and whose magnitude is .12 teslas. If the magnetic field drops to .04 teslas in 2.4 seconds:

a.) What is the induced EMF in the loop during the drop?

b.) What is the direction of the loop's current during the drop?

c.) If the current in the loop is found to be .15 amps during the drop, what is the resistance in the loop?

19.5)   For the RL circuit shown in Figure III, the inductance is 1.5 henrys and the inductor's internal resistance is 6 ohms. A current of 2.5 amps has been flowing in the circuit for a long time. At t = 0, the power is switched off and the current begins to die.

a.) What is the voltage across the inductor BEFORE t = 0?

b.) After .05 seconds, the current has dropped to approximately one-third of its original value. Determine the resistance of the resistor R. (Hint: think about the time constant of an RL circuit and what it tells you).

c.) How much POWER does the inductor provide to the circuit over the .05 second time period alluded to in Part b? (Hint: Think about the definition of power and what you know about stored energy in a current-carrying inductor).
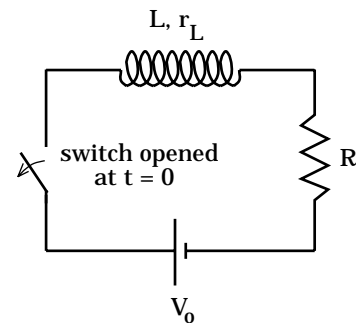
d.) The power given up by the inductor: where did it go?

FIGURE III

19.6) A closed, single-turn, rectangular loop of length h = 1.2 meters, width w = .6 meters, mass m = .07 kilograms, and resistance R = 15 ohms

initially sits stationary in a bounded, uniform magnetic field whose magnitude is .85 teslas (see Figure IV). The loop is released and begins to accelerate downward due to gravity, exiting the field in the process.

B-field into page

FIGURE IV

     a.) In what direction will the induced current in the coil move?

     b.) Terminal velocity is defined as the velocity at which the net force on a body, and hence the body's acceleration, goes to zero. Though we will be ignoring air friction, terminal velocity occurs in this case. That is, at some velocity the gravitational force will be equal and opposite the magnetic force produced by the interaction of the induced current in the loop and the magnetic field. Your thrill is to determine that terminal velocity. (Hint: Begin by determining the current in the loop when terminal velocity has been reached; with that, determine the magnetic force on the current-carrying wire and use N.S.L. to finish up).

     c.) Derive an expression for the coil's velocity as a function of time. You might find it useful to note that $\int_{c}^{d} \dfrac{dv}{(a+bv)} = \dfrac{1}{b} \ln(a+bv) \big|_{c}^{d}$.

19.7) A rectangular coil of area A_0 has N turns in it. It is rotated in a time-varying magnetic field (see Figure V) equal to $B_0 e^{-kt}$, where k is a constant and B_0 is the amplitude of the magnetic field.

B-field out of page

FIGURE V

Assuming the frequency of the rotation is v:

     a.) Determine the EMF in the coil as a function of time, and;

     b.) At what point in time will the magnitude of the EMF be at maximum?

19.8) A fixed circular coil of radius R is placed in a magnetic field that varies as $12t^3 - 4.5t^2$. If the coil has N winds and A is defined out of the page (i.e., in the +k-direction):

field direction is time-dependent--the direction of A is out of the page

FIGURE VI

     a.) Is B into or out of the page at t = .2 seconds?

     b.) Derive a general expression for the magnetic flux through the coil.

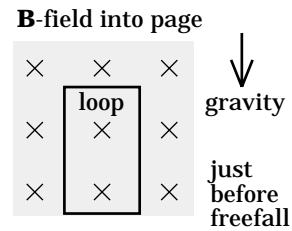   c.) What is the general expression for the induced EMF in the coil?
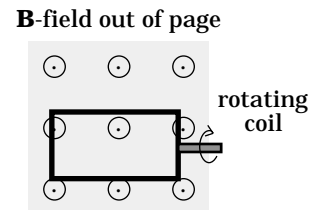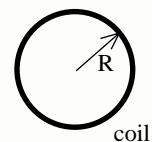
d.) Determine the two points in time when the induced EMF is zero.

e.) What is the direction of the current flow:

i.) Just before t = .25 seconds?

ii.) Just after t = .25 seconds?

f.) Derive the general expression for the induced electric field setup in the coil.

g.) An electron is placed at R/2 in the field. Derive an expression for its acceleration at time t = 3.3 seconds. For this, assume N = 15 and R = .2 meters.

19.9) THIS IS AN IMPORTANT PROBLEM TO UNDERSTAND!!! A magnetic field is set up by a current-carrying wire. A fixed aluminum frame is placed in the field and a crossbar of mass m is made to move toward the wire with a constant velocity v (see Figure VII). If the electrical resistance in the circuit is R, and assuming the bar stays in frictionless contact with the frame at all times:
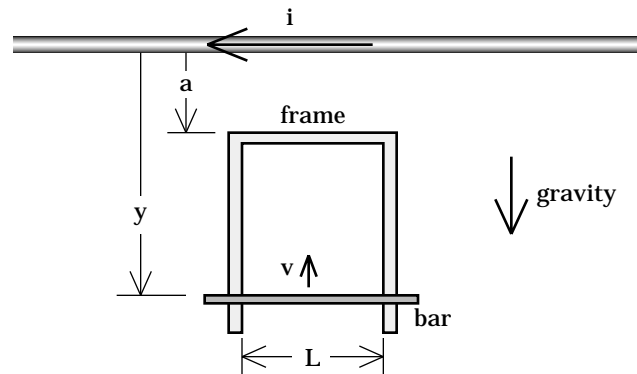

FIGURE VII

a.) Derive a general expression for the EMF induced in the system as the bar moves.

b.) How much force must YOU supply to the system to keep the bar moving with constant velocity v?

19.10) The transformer shown in Figure VIII has 1200 winds in its primary coil and 25 winds in its secondary. The resistance in its primary is 80 Ω's, the resistance in its secondary is 3 Ω's, and the primary's inductance is L_p = 10 mH. A 110 volt DC power supply is hooked into the primary providing an 8.25 amp current to the system. The switch has been closed for a long time. When the switch is opened, the current drops to zero in .04 seconds.
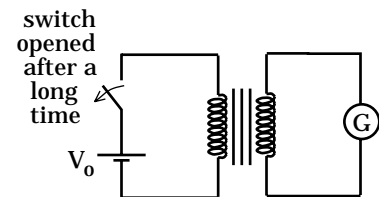

FIGURE VIII

a.) What is the induced EMF across the primary before the switch is opened?

b.) What is the induced EMF across the primary during the current change?

c.) What is the current in the primary during the current change (i.e., after the switch is opened)?

d.) What is the current in the secondary before the switch is opened?

e.) What is the current in the secondary after the switch is opened and DURING the current change?

f.) Is this a step-up or step-down transformer?